Public Economics (ECON 131)
Section #11: Moral Hazard and Social Security (continued)

April 17 and 19, 2017

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1 Key Concepts

See last week. Additionally:

- Social Security
  - What is the **most important rationale** for social security?
- Moral hazard
  - Why does the result that full insurance is optimal **fail** under moral hazard?

2 Practice Problems

2.1 Moral Hazard

A firm has to decide an insurance plan to provide for a single worker. The worker has probability $q = 0.2$ of becoming unemployed, and $1 - q = 0.8$ of staying employed. The worker can choose his unemployment duration, D, to be maximum 1 year, i.e. $D \in [0, 1]$.

If the worker stays employed he gets an income of $w = 100$ minus the insurance premium p. If he loses his job he gets the insurance benefit bD for the part of the year that he is unemployed, and $w(1-D) = 100(1-D)$ for the part of the year that he is employed. In addition, he gets disutility from searching for a job. Since searching less for a job implies longer unemployment durations, this is the same as saying that he gets positive utility from a longer unemployment duration, D. With an instantaneous utility function $u(c) = \ln(c)$ this means that expected utility is

$$EU = 0.8 \ln(100 - p) + 0.2(\ln(bD + 100(1 - D) - p) + D) \tag{1}$$

where $\ln(100 - p)$ is his utility under employment and $\ln(bD + 100(1 - D) - p) + D$ is his utility under unemployment.

(a) *What is the benefit level as a function of the premium under an actuarially fair insurance program?*

Solution:

Under an actuarially fair program,

$$\text{Expected profits} = p - 0.2bD = 0$$

so $b = \frac{p}{0.2D}$

(b) *The insurance company expects that the worker will be unemployed for half a year on average, i.e. $D = 0.5$. Which premium and benefit level ensures full insurance under the actuarially fair insurance program in the previous question? Call this Policy #1.*

Solution: For full insurance we need

$$\begin{aligned} 100 - p &= bD + 100(1 - D) - p \\ 100 - p &= \frac{p}{0.2} + 50 - p \\ 100 - p &= \frac{0.8}{0.2}p + 50 \\ p &= 10 \end{aligned}$$

so the corresponding benefit level is $b = \frac{10}{0.2 \cdot 0.5} = 100$

(c) *It turns out that the insurance company's guess about the unemployment duration length might not have been correct. Write up an expression for the worker's utility under Policy #1, i.e. plug b and p into (1). Which D will the worker actually choose? What is his utility under Policy #1?*

Solution:

$$\begin{aligned} EU &= 0.8\ln(100-p) + 0.2(\ln(bD + 100(1-D) - p) + D) \\ &= 0.8\ln(100-10) + 0.2(\ln(100D + 100(1-D) - 10) + D) \\ &= 0.8\ln(90) + 0.2(\ln(90) + D) \\ &= \ln(90) + 0.2D \end{aligned}$$

Since utility is increasing in D, they will set D at its maximum, i.e. $D = 1$. Hence, his utility will be $ln(90) + 0.2 = 4.6998$

(d) *The firm learns that it was losing money under Policy #1. Hence, it proposes a new actuarially fair insurance plan with full insurance under the actual D. Call this policy #2. What is the new premium p and benefit b? What is the worker's utility under this policy? Compare it to his utility under policy #1. Would he in hindsight still have chosen to change his unemployment duration rather than keep it at $D = 0.5$?*

Solution: Under full insurance

$$\begin{aligned} 100 - p &= b - p \\ b &= 100 \end{aligned}$$

so $p = 0.2 \cdot 100 = 20$. His utility will be

$$\begin{aligned} EU &= 0.8\ln(100-p) + 0.2(\ln(bD + 100(1-D) - p) + D) \\ &= 0.8\ln(100-20) + 0.2(\ln(100D + 100(1-D) - 20) + D) \\ &= 0.8\ln(80) + 0.2(\ln(80) + D) \\ &= \ln(80) + 0.2D \end{aligned}$$

So his utility will be $\ln(80) + 0.2 = 4.58$, i.e. he would rather have kept a duration of $D = 0.5$.

(e) *Now, suppose the firm implements a new plan (Policy #3) under which full insurance is* not *provided. In particular the firm provides a benefit of $b = 20$. Under this lower benefit the firm expects a low unemployment duration, namely $D = 0.25$. Under an actuarially fair program this means that $p = 1$ (check this!).*

- *(i) Which unemployment duration will the worker actually choose?*
- *(ii) Will the firm lose money under this D?*
- *(iii) Compute the worker's utility. Which of the three policies does he provide?*
- *(iv) Why does your answer differ from the result in class that full insurance is optimal under no behavioral responses?*

Solution: The worker's utility is now

$$\begin{aligned} EU &= 0.8\ln(100-1) + 0.2(\ln(20D + 100(1-D) - 1) + D) \\ &= 0.8\ln(99) + 0.2(\ln(99 - 80D) + D) \end{aligned}$$

Maximizing with respect to D,

$$\begin{aligned} \frac{80}{99-80D} &= 1 \\ D &= \frac{19}{80} \end{aligned}$$

Since $\frac{19}{80} < 0.25$, the firm actually gets a profit under this policy.

The worker's utility is

$$\begin{aligned} EU &= 0.8\ln(99) + 0.2(\ln(99 - 80\frac{19}{80}) + 0.2\frac{19}{80}) \\ &= 0.8\ln(99) + 0.2(\ln(80) + 0.2\frac{19}{80}) \\ &= 4.6 \end{aligned}$$

So he will choose policy #3. The reason why he won't choose full insurance is because under full insurance he can't commit to keeping a short unemployment duration. This implies that the insurance company will need to provide an expensive insurance plan in order to break even. However, under less-than-full insurance he will be able to commit to a low unemployment duration, so the insurance plan will be cheap, and he will get a higher utility.

2.2 Social Security

Consider the following model of social security. N people are born each period. Each person lives for two periods. In the first period of life, a person is young and in the second old. Thus, in any period after the initial period, half the population is young and half is old. Young people earn 2 chocolate bars while old people earn nothing. Assume that the chocolate melts, so there is no way for people to save privately.

(a) *Give a brief (one sentence) rationale for government provision of social security in this model.*

Solution: Individuals cannot save privately since the chocolate melts so the government must provide social security so that they can eat when they are old.

Now suppose that the government of Candyland implements a social security system in the following manner. The government taxes each young person 1 chocolate bar and redistributes it to an old person in the same period. The program starts between periods 0 and 1 and ends between periods 2 and 3, as shown below. Let c_A^g denote the chocolate consumption of an agent from generation g (generation refers to the period the agent is born) at age A (age is either young or old).

(b) *Under Candyland's social security program, fill in the blanks in the following chart:*

Period 0		Period 1	Period 2		Period 3
	Start SS			End SS	
$c^0_{young} = \_\_$	$\vert$	$c^0_{old} = \_\_$		$\vert$	
	$\vert$	$c^1_{young} = \_\_$	$c^1_{old} = \_\_$	$\vert$	
	$\vert$		$c^2_{young} = \_\_$	$\vert$	$c^2_{old} = \_\_$

Which generation gains the most in terms of consumption? The least?

Solution:

Period 0		Period 1	Period 2		Period 3
	Start SS			End SS	
$c^0_{young} = 2$	$\vert$	$c^0_{old} = 1$		$\vert$	
	$\vert$	$c^1_{young} = 1$	$c^1_{old} = 1$	$\vert$	
	$\vert$		$c^2_{young} = 1$	$\vert$	$c^2_{old} = 0$

Generation 0 benefits most, being able to enjoy the benefits of the social security system without having to incur the costs of paying for a previous generation's consumption. On the other hand, generation 2 loses the most, since they pay the tax but never enjoy the benefits of the system.

(c) *What assumption does this model make about the effect of social security provision on retirement behavior? Discuss how empirical evidence on retirement decisions and social security relates to this assumption.*

Solution: The model assumes that the fraction of people who are "old" is invariant to the social security system. In other words, it assumes that retirement behavior is not affected by the provision of SS. In contrast, Gruber and Wise (2002) present evidence on retirement behavior around the world, and find that workers's retirement decisions are heavily influenced by whether or not additional work will increase their social security benefits.

From lecture: Gruber and Wise (1999) calculated the implicit tax (on work) from Social Security for a series of countries.

- Across countries, there is a great deal of variation in the implicit tax rate.
  - Implicit tax close to zero for 62-year-olds in the United States.
  - 91% in the Netherlands.
- And countries with higher taxes have less elderly labor force participation.

(d) *The country of Twixland is also considering implementing social security. However, the ingenious residents of Twixland have figured out a way to freeze and save chocolate for retirement. All of them have utility over consumption when young (c_A) and old (c_B) given by $\log(c_A) + \log(c_B)$. The demographic and other aspects of the economy are as in Candyland.*

Replicate the chart in (b) for Twixland, assuming that the start and end of SS are completely unanticipated by its residents.

Solution: Since the utility is concave, individuals will want to smooth consumption perfectly across generations consume 1 chocolate bar when they are young and 1 when they are old. In Twixland, this is feasible in the absence of SS because people can save.

Twixlanders react to the new SS program by changing their savings decisions accordingly if they anticipate the receipt of SS when they are old. Generation 0 individuals do not anticipate getting SS when old, so they continue to save 1 chocolate bar in period 0, but gain an extra chocolate bar in period 1 because of SS. Generation 1 anticipates SS, so its consumption is unchanged (100% crowdout of government intervention). Generation 2 anticipates getting SS and therefore does not save for itself, but then gets nothing in period 3.

Period 0		Period 1	Period 2		Period 3
	Start SS			End SS	
$c^0_{young} = 1$	|	$c^0_{old} = 2$		|	
	|	$c^1_{young} = 1$	$c^1_{old} = 1$	|	
	|		$c^2_{young} = 1$	|	$c^2_{old} = 0$

Again, we see that generation 0 benefits the most and generation 2 loses. The only difference relative to CandyLand is that generation 0 gains more consumption when old rather than young.